1. [N20 Paper 2 #5]

(a) Assuming the Maclaurin series for $\cos x$ and $\ln(1+x)$, show that the Maclaurin series for $\cos(\ln(1+x))$ is [4]

$$1 - \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{5}{12}x^4 + \dots$$

(b) By differentiating the series in part (a), show that the Maclaurin series for $\sin(\ln(1+x))$ is $x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$ [4]

(c) Hence determine the Maclaurin series for $\tan(\ln(1+x))$ as far as the term x^3. [5]

2. [N21 Paper 1 #11]

(a) Prove by mathematical induction that $\dfrac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x^2e^x\right) = [x^2 + 2nx + n(n-1)]e^x$ for $n \in \mathbb{Z}^+$. [7]

(b) Hence or otherwise, determine the Maclaurin series of $f(x) = x^2e^x$ in ascending powers of x, up to and including the term in x^4. [3]

(c) Hence or otherwise, determine the value of $\lim\limits_{x\to 0}\left[\dfrac{(x^2e^x - x^2)^3}{x^9}\right]$.

3. [M21 TZ2 Paper 2 #9]

(a) Write down the first three terms of the binomial expansion of $(1+t)^{-1}$ in ascending powers of t. [1]

(b) By using the Maclaurin series for $\cos x$ and the result from part (a), show that the Maclaurin series for $\sec x$ up to and including the term in x^4 is $1 + \frac{x^2}{2} + \frac{5x^4}{24}$. [4]

(c) By using the Maclaurin series for $\arctan x$ and the result from part (b), find $\lim_{x \to 0} \left(\frac{x \arctan 2x}{\sec x - 1} \right)$. [3]

4. [M21 TZ1 Paper 1 #12]

Let $f(x) = \sqrt{1+x}$ for $x > -1$.

(a) Show that $f''(x) = -\dfrac{1}{4\sqrt{(1+x)^3}}$. [3]

(b) Use mathematical induction to prove that $f^{(n)}(x) = \left(-\dfrac{1}{4}\right)^{n-1} \dfrac{(2n-3)!}{(n-2)!}(1+x)^{\frac{1}{2}-n}$ for $n \in \mathbb{Z}$, $n \geq 2$. [9]

Let $g(x) = e^{mx}$, $m \in \mathbb{Q}$.

Consider the function h defined by $h(x) = f(x) \times g(x)$ for $x > -1$.

It is given that the x^2 term in the Maclaurin series for $h(x)$ has a coefficient of $\dfrac{7}{4}$.

(c) Find the possible values of m. [8]

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5. [Specimen Paper 1 # 12]

The function f is defined by $f(x) = e^{\sin x}$.

(a) Find the first two derivatives of $f(x)$ and hence find the Maclaurin series for $f(x)$ up to and including the x^2 term. [8]

(b) Show that the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero. [4]

(c) Using the Maclaurin series for $\arctan x$ and $e^{3x} - 1$, find the Maclaurin series for $\arctan\left(e^{3x} - 1\right)$ up to and including the x^3 term. [6]

(d) Hence, or otherwise, find $\lim\limits_{x \to 0} \dfrac{f(x) - 1}{\arctan(e^{3x} - 1)}$.